Find the 5^{th} term in the expansion of $(7n^2 - 3n^5)^{27}$. **SCORE: _____ / 5 PTS**

NOTES: **<u>Your coefficient may use the operations +, –, × and positive exponents only.</u>**
<u>All exponents must be simplified and all signs must be explicit (at the front).</u>

$${}_{27}C_4(7n^2)^{27-4}(-3n^5)^4 = {}_{27}C_4(7n^2)^{23}(-3n^5)^4$$

$$= \frac{27!}{4!\,23!}7^{23}n^{46}(-3)^4n^{20}$$

1 POINT EACH

SUBTRACT 1 POINT IF YOUR ANSWER IS NEGATIVE

$$= \boxed{\frac{27\cdot 26\cdot 25\cdot \cancel{24}\cdot \cancel{23!}}{\cancel{4\cdot 3\cdot 2}\cdot 1\cdot \cancel{23!}}}\,7^{23}3^4n^{66}$$

$$= \boxed{27\cdot 26\cdot 25}\cdot\boxed{7^{23}}\cdot\boxed{3^4}\boxed{n^{66}}$$

Use the entries of Pascal's Triangle to expand and simplify $(9t - 10z)^5$. **SCORE: _____ / 5 PTS**

NOTES: **<u>Your coefficients may use the operations +, –, × and positive exponents only.</u>**
<u>All exponents must be simplified and all signs must be explicit (at the front).</u>
<u>You must show the intermediate step in the expansion to get full credit.</u>

$$1(9t)^5(-10z)^0 + \boxed{5(9t)^4(-10z)^1} + \boxed{10(9t)^3(-10z)^2} + \boxed{10(9t)^2(-10z)^3}$$
$$+\boxed{5(9t)^1(-10z)^4} + 1(9t)^0(-10z)^5$$

$$= \boxed{9^5t^5}\boxed{-5\cdot 9^4\cdot 10t^4z}\boxed{+10\cdot 9^3\cdot 10^2t^3z^2}\boxed{-10\cdot 9^2\cdot 10^3t^2z^3}$$
$$\boxed{+5\cdot 9\cdot 10^4tz^4}\boxed{-10^5z^5}$$

$\frac{1}{2}$ POINT EACH ☆ SUBTRACT 1 POINT IF YOUR NEGATIVES IN THE FINAL ANSWER ARE INSIDE ( )'S

Find the rational number representation of the repeating decimal $0.6\overline{54}$. **SCORE: _____ / 5 PTS**

NOTES: **<u>Only the 54 is repeated.</u>**
<u>You must use only techniques from sections 9.2-9.5.</u>

$$\boxed{0.6 + 0.054 + 0.00054 + 0.0000054 + \dots} \quad 1$$

$$= \frac{6}{10} + \boxed{\frac{\frac{54}{1000}}{1-\frac{1}{100}}} \quad 1\tfrac{1}{2}$$

$$= \frac{3}{5} + \frac{\frac{54}{1000}}{\frac{99}{100}}$$

$$= \frac{3}{5} + \frac{\overset{3}{\cancel{54}}}{{}_{5}\cancel{1000}}\cdot\frac{\cancel{100}}{\underset{11}{\cancel{99}}}$$

$$\tfrac{1}{2}\ = \boxed{\frac{3}{5}} + \boxed{\frac{3}{55}} \quad 1$$

$$= \frac{33+3}{55} = \boxed{\frac{36}{55}} \quad 1$$

AJ and BJ both have sequences where the third term is 81, and the sixth term is -24. **SCORE: _____ / 5 PTS**

[a] If AJ's sequence is geometric, find the first term of AJ's sequence.

$a_3 = \boxed{a_1 r^2 = 81}$
$a_6 = \boxed{a_1 r^5 = -24}$ $\frac{1}{2}$ $\longrightarrow \frac{a_1 r^5}{a_1 r^2} = \frac{-24}{81} \longrightarrow \boxed{r^3 = -\frac{8}{27}}$ $\frac{1}{2}$ $\longrightarrow \boxed{r = -\frac{2}{3}}$ $\frac{1}{2}$

$a_1\left(-\frac{2}{3}\right)^2 = 81 \longrightarrow \frac{4}{9}a_1 = 81 \longrightarrow \boxed{a_1 = \frac{729}{4}}$ $\frac{1}{2}$

[b] If BJ's sequence is arithmetic, find the sum of the first 11 terms of BJ's sequence.

$a_3 = \boxed{a_1 + 2d = 81}$ $\frac{1}{2}$ $a_1 + 2(-35) = 81$
$a_6 = \boxed{a_1 + 5d = -24}$ $\frac{1}{2}$ $a_1 = 151$

$\boxed{-3d = 105}$ $\frac{1}{2}$ 1 $S_{11} = \boxed{\frac{11}{2}(2(151) + (11-1)(-35))}$ $\frac{1}{2}$

$\boxed{d = -35}$ $\frac{1}{2}$ $= \frac{11}{2}(302 - 350) = \frac{11}{\cancel{2}} \cdot \underset{-24}{\cancel{-48}} = \boxed{-264}$

Using mathematical induction, prove that $\sum_{i=1}^{n}[(2i+1)\cdot 3^i] = n \cdot 3^{n+1}$ for all positive integers n. **SCORE: _____ / 10 PTS**

BASIS STEP: PROVE $\sum_{i=1}^{1}(2i+1)\cdot 3^i = 1 \cdot 3^{1+1} = 1 \cdot 3^2$

$\sum_{i=1}^{1}(2i+1)\cdot 3^i = (2(1)+1)\cdot 3^1 = 3 \cdot 3 = 9$

$1 \cdot 3^2 = 9$ ✓

INDUCTIVE STEP: ASSUME $\sum_{i=1}^{k}(2i+1)\cdot 3^i = k \cdot 3^{k+1}$ FOR SOME PARTICULAR BUT ARBITRARY INTEGER $k \geq 1$

PROVE $\sum_{i=1}^{k+1}(2i+1)\cdot 3^i = (k+1)\cdot 3^{(k+1)+1} = (k+1)\cdot 3^{k+2}$

$\sum_{i=1}^{k+1}(2i+1)\cdot 3^i = \sum_{i=1}^{k}(2i+1)\cdot 3^i + (2(k+1)+1)\cdot 3^{k+1}$

$= k \cdot 3^{k+1} + (2k+3)\cdot 3^{k+1}$

$= (k + 2k + 3)\cdot 3^{k+1}$

$= (3k+3)3^{k+1}$

$= 3(k+1)3^{k+1}$

$= (k+1)3^{k+2}$

GRADED BY ME

BY MI, $\sum_{i=1}^{n}(2i+1)\cdot 3^i = n \cdot 3^{n+1}$ FOR ALL POSITIVE INTEGERS n